ТЕОРЕТИЧЕСКАЯ ФИЗИКА

THERMODYNAMICS OF THE QCD CROSSOVER AT FINITE TEMPERATURE AND ZERO CHEMICAL POTENTIAL WITHIN THE PNJL MODEL FOR A QGP WITH TWO LIGHT QUARK FLAVORS

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We explore the Quantum Chromodynamics (QCD) phase transition using an approach where the hadronic phase is described at low-temperature, while the high-temperature Quark–Gluon Plasma (QGP) phase is represented by the Polyakov–Nambu–Jona-Lasinio (PNJL) model. The two phases are connected through a switching function ensuring the continuity of the thermodynamic quantities and a smooth transition between the two regimes. The analysis focuses on the pressure and the squared speed of sound, as functions of temperature, at zero chemical potential. We test the influence of the pion mass in the hadronic phase, and we examine the effect of the Vandermonde (VdM) term, which is often added to the Polyakov effective potential to constrain the Polyakov loop within its physical domain. Our results show that the pion mass induces a moderate shift of the crossover region and improves agreement with lattice QCD data near the transition. Neglecting the VdM term does not significantly affect the thermodynamic quantities near the transition temperature. These findings provide new quantitative insights into how hadronic mass effects and Polyakov loop dynamics influence the QCD crossover region.

Keywords: QCD phase transition, Switching function, Polyakov loop, Vandermonde term, Lattice QCD.

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Introduction. Understanding the Quantum Chromodynamics (QCD) phase transition from hadronic matter to the Quark-Gluon Plasma (QGP) at high temperatures and/or baryon densities is a key problem in particle and nuclear physics, as indicated by lattice QCD simulations [1]. The QGP state is believed to have existed during the first microseconds after the Big Bang and can be recreated only through heavy-ion collisions [2]. Lattice QCD studies [3] reveal a smooth hadron–QGP crossover at high temperatures and low chemical potentials. Lattice QCD effectively studies QCD thermodynamics but is limited at finite chemical potential due to the sign problem [4]. To study the QCD phase diagram, various effective models have been developed, including the Nambu–Jona-Lasinio (NJL) model [5], linear sigma models [6], while the Polyakov–Nambu–Jona-Lasinio (PNJL) model [7, 8] accounts for confinement–deconfinement effects through the Polyakov loop fields.

In QCD, the equation of state describes nuclear matter transitioning from the confined hadronic phase to the deconfined QGP phase. A switching function can smoothly interpolate between the two phases, providing a physically motivated crossover consistent with QCD predictions and heavy-ion collision observations [9].

In this work, The connection between the hadronic matter and the QGP is implemented using the switching function within the equation of state to ensure the continuity of physical quantities, as in [8]. We investigate the thermodynamic behavior of the strongly interacting matter in a mixed HG-QGP system essentially by calculating two key thermodynamic quantities, namely the pressure and the squared speed of sound, and examine their behavior as functions of temperature, at vanishing chemical potential. We focus on the influence of the pion mass in the hadronic phase and the Vandermonde term in the PNJL model.

Statistical description of the hadronic phase. In the following, we aim to compute the partition function Z_{HG} of the hadronic gas phase, consisting of the pions $(\pi^0, \pi^{\pm})$, which are the lightest mesons, with a mass: $m_\pi = 138$ MeV. The Bose-Einstein statistics allows writing...

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